Modeling Human Perception

Could Stevens' Power Law be an Emergent Feature?

Matthew L. Bolton
Systems and Information Engineering
University of Virginia
Charlottesville, United States of America
Mlb4b@Virginia.edu

***Abstract*— Stevens' power law links the magnitude of a physical stimulus to its perceived internal intensity via a psychophysical power function. Because of the law's sensitivity to the procedure used to collect its data, its failure to manifest itself at the individual level, its manifestation in non-sensory modalities, and the difficulty associated with rating stimuli on a ratio scale, many have speculated that the power law is a product of the experimental procedure Stevens used. This work tested this hypothesis by reproducing one of Stevens' original power law experiments (using different pressure levels of a 1 kHz tone) in which 52 data series were generated from a computerized process that simulated Stevens' experimental procedure. However, instead of generating ratio data for each magnitude estimate, random ordinal data were generated. When a power function was fitted to these data using the same procedure utilized by Stevens, it fit with an adjusted R^2 of 0.997, while generating the same psychophysical model. This indicates that Stevens' assumption that participants make magnitude estimates on a ratio scale is not necessary to collect power law data. Thus, Stevens' power law is likely a product of Stevens' experimental procedure.**

***Keywords*—Stevens Power Law, Psychophysics, Perception**

I. Introduction

Stevens' power law [1] is a psychophysical law that links the intensity of a physical stimulus to its perceived internal intensity (sensation) on a ratio scale. It is commonly represented as $\psi(I) = kI^a$ where:

- I is the magnitude of the physical stimulus,
- ψ is a psychometric function that maps I into sensation,
- k is a constant that is dependent on both the sensory modality and units of the physical stimulus (as well as other factors), and
- a is constants that depends on the sensory modality.

Through extensive experimentation, it has been found that perceptual modalities of prothetic qualities (qualities that encompass quantitative information such as loudness, brightness, and pressure) often have a unique exponent (a), that determines the slope of the power function in log-log coordinates (see Figure 1 for several examples).

A. Magnitude Estimation

Stevens [1] outlined an experimental procedure called magnitude estimation for the purpose of collecting power law data. At the start of the experiment, a stimulus is presented as a standard (with either a researcher or participant assigned subjective numerical rating – a magnitude estimate). Then,

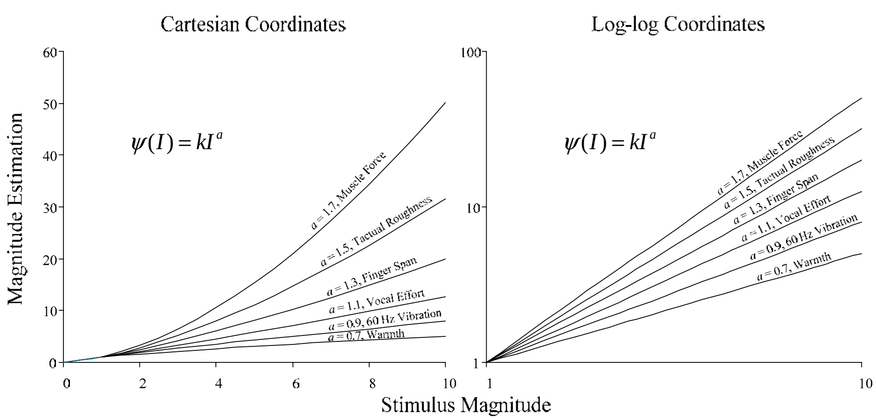

Figure 1. .Power law exponents for six different perceptual modalities on both Cartesian and log-log coordinates [2].

1-4244-2384-2/08/$20.00 © 2008 IEEE

sequences of variable stimulus magnitudes (usually sampled from a geometric sequence) are presented, with the participant providing a numerical magnitude estimate of the intensity of each with respect to the standard on a ratio scale. For example, if a participant perceives a stimulus as being twice as intense as the standard, it would be given a magnitude estimate twice as large as the standard's magnitude estimate. This entire procedure is often performed twice (two sessions) for a single participant with the same stimulus magnitude levels, but with a different standard and a different presentation order for the variable stimulus magnitudes.

Once all of the data are collected, they are scaled using a procedure Stevens called modulus equalization. To do this, each participant's magnitude estimates (judgments) are multiplicatively scaled so that the geometric mean of his or her estimates for a given stimulus magnitude are equal to a given number (Stevens usually scaled the estimates for a median stimulus magnitude to 10).

Further, Stevens [1] provided the following procedural recommendations to help researchers collect power law data:

1. Use a standard near the center of the physical stimulus's range (not at the perceptual extremes);
2. Present variable stimuli on either side of the standard;
3. Call the standard by a number (Stevens usually used 10);
4. Allow the participant to determine all other numerical assignments beyond the standard;
5. Use only one standard level per experimental session;
6. Randomize the variable stimuli presentation order;
7. Keep experimental sessions short;
8. Let the participant control stimuli presentation; and
9. Use groups of participants large enough to account for high variability in results.

If these procedures are followed, researchers are likely to obtain a power curve with a good statistical fit.

To illustrate the magnitude estimation procedure we examine one of Stevens' foundational power law experiments [1][3]. For a given experimental session, participants were presented with eight different intensities (sound pressure levels) of a 1 kHz tone from a geometric series (0.2, 2, 20, 200, 2000, 20000, 200000, and 2000000 Pa corresponding to dB levels from 40 to 110 in 10 dB intervals) in a random order. The first stimulus a participant heard was the standard, which was assigned a magnitude estimate of 10. Participants were instructed to provide magnitude estimates for each of the subsequent stimuli by providing a ratio estimate of its loudness with respect to the standard. This procedure was performed twice for each of 26 participants. When modulus equalization was carried out across participants (scaling each participant's magnitude estimates so that the average estimate for the 80 bB stimulus was 10) and the power curve was fit to the median estimates for each stimulus, an exponent of 0.3 was observed (Figure 2).

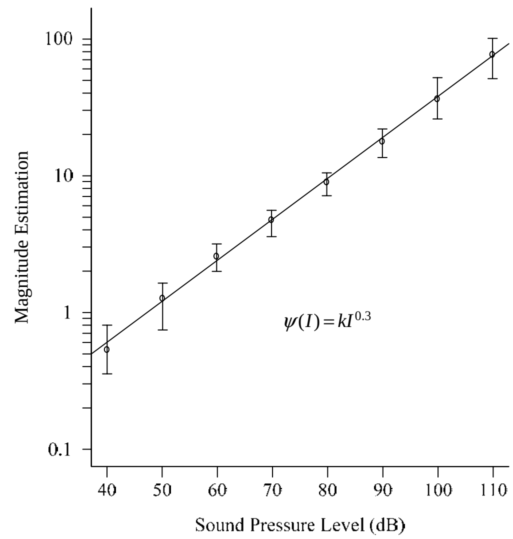

Figure 2. Power law functions obtained from an experiment with 26 participants for different intensities (sound pressure levels) of a 1 kHz tone (re-plotting of data from [1] & [3]). Circles indicate medians and bars indicated interquartile ranges.

B. Stevens' Power Law as an Emergent Feature

Despite its acceptance in the psychological community, there is evidence that suggests Stevens' power law is an emergent feature of the magnitude estimation procedure. This includes all of the following: the inability of experimenters to obtain power law data using modified experimental procedures in which one or more of Stevens' recommendations were not adhered to [4]; the failure of the law to consistently manifest at the participant level [5]; and the ability to successfully derive power law data from non-sensory modalities such as perceived numerousness, length, and duration [4].

Criticism has also focused on Stevens' assertion that participants' magnitude estimations were being made on a ratio scale. Narens [6] formulated two testable hypotheses based on this assumption. First, power law estimates would have a multiplicative property: for three given stimulation magnitudes x, y, and z and two proportions p and q, if someone judges x as p times more intense than y, and y as q times more intense than z, and then he or she should judge x as pq times more intense than z:

$$x = py \wedge y = qz \to x = pqz$$

Second, power law estimates would have a commutative property: for stimulation magnitudes t, w, x, y, and z and proportions p and q, if someone judges x as p times more intense than t, z as q times more intense than x, y as q times more intense than t, and w as p times more intense than y, then z = w:

$$x = pt \wedge z = qx \wedge y = qt \wedge w = py$$
$$\rightarrow z = pqt \wedge w = pqt \rightarrow z = w$$

Ellermeier and Faulhammer [7] and Zimmer [8] found that participant magnitude estimates violated the multiplicative property, but not the commutative property. As Narren [6] acknowledges, this is sufficient for magnitude estimates to be made on a ratio scale, but not as powerful as it would be if the multiplicative property held.

Stevens [1] acknowledged that some participants had trouble producing ratio data during magnitude estimation. Those having trouble were able to produce ordinal data.

C. *Objectives*

This work sought to determine if purely ordinal magnitude estimates made in the context of Stevens' magnitude estimation procedure would be capable of producing power law data compatible with those found in Stevens' investigation of the 1 kHz tone. This was done using a simulation where a process imitated humans making random ordinal judgments in the context of Stevens' magnitude estimation procedure.

II. METHODS

This work constructed a simulation (an R script) that mimicked the procedure Stevens used to gather power law data. However, in place of collecting a magnitude estimation from a participant for given stimulus magnitude, a simulated participant generated a random number based on the ordinal relationships between stimuli. Multiple data series were generated using this simulation and were evaluated using the same statistical procedures used by Stevens.

For a given experimental session, the simulation generated a random experimental order of stimuli from a geometric sequence. Because the first stimulus in an order served as the standard (as was done by Stevens, [1]), random orders were constrained to ensure that only the middle third of the stimuli magnitude were presented first. This complied with Stevens' procedural recommendation that standards should not be at the perceptual extremes of the continuum being evaluated. This also facilitated an additional recommendation that stimuli be presented in a random order on either side of the standard.

With an established random order, random ordinal judgments were generated by mimicking Stevens' [1] procedure. The process treated the first stimulus magnitude as the standard and assigned it the number 10 (as was Steven's standard practice). This value was recorded as the magnitude estimate for the standard stimulus. Magnitude estimates for the remaining stimuli were generated sequentially (based on their experimental order) as follows:

1. If a stimulus was smaller than any stimulus presented previously in the order, its magnitude estimate was assigned a random number from a uniform distribution between zero (non-inclusively) and the magnitude estimate given for the previously minimal stimulus.
2. If a stimulus fell between two previously presented stimuli, its magnitude estimate was assigned a random number from a uniform distribution between the magnitude estimates of the surrounding stimuli (the closest neighbors on either side of the presented stimulus).
3. If a stimulus was larger than any stimulus presented previously, then its magnitude estimate was assigned a random number from a uniform distribution between the magnitude estimate for the previous maximum and the magnitude estimate for the previous maximum increased by a constant increment.

This procedure had the effect of simulating ordinal judgment values that, while constrained by zero as its lower bound, could grow large for progressively larger stimuli. Zero was used as the lower bound to mimic the effect of Stevens' instructions for participants to provide responses based on the stimulus's multiplicative relationship to the standard [1]. Since negative and zero estimates do not make sense in this context, they were not allowed in this simulation.

The simulation completed by performing modulus equalization (it assumed that two concurrent sessions were performed by a single participant). The medians of the resulting data were computed for each stimulus intensity and a power curve was fit to these using least squares regression. The medians and fitted line were then plotted on a log-log plot with the interquartile range of the data for each stimulus.

The simulation model had three parameters used to fit a specific psychophysical power function. First, the geometric sequence of stimuli intensities could be changed in order to match those used in power law experiments. Second, the number of simulated participants in the experiment could be changed to match the number of participants from in given experiment. Third, the constant increment used to control the magnitude estimation of a maximal stimulus (see step 3 above) could also be changed.

In this work, the geometric sequence of stimulus magnitudes was 0.2, 2, 20, 200, 2000, 20000, 200000, and 2000000 Pa corresponding to dB levels from 40 to 110 in 10 dB intervals (the same values used by Stevens [1][3]). Fifty two data series were generated, matching Stevens' 26 participants (two data series for each). The initial simulation was run with a constant increment of 100.

III. RESULTS

Using the initial set of parameters the simulation was able to produce power law data. In this case, the model $\psi(I) = (1.473)I^{0.243}$ significantly predicted magnitude estimation while explaining a significant proportion of the variance (Figure 3). Further, through iterative increases in the increment, an exponent of 0.301 (closely matching the 0.3 exponent reported by Stevens) was achieved using an increment of 200 to produce a model $\psi(I) = (0.977)I^{0.301}$, which significantly predicted magnitude estimation and explained a significant proportion of the variance (Figure 4).

When these results are compared directly with those obtained by Stevens (Figure 5), it is clear that the simulation was capable of replicating the median estimates of the participants in Stevens' experiment. However, it is important to note that the simulation results did produce slightly higher

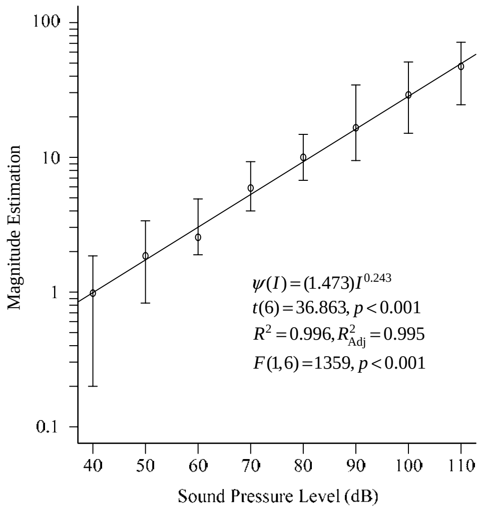

Figure 3. Simulation generated results using an increment of 100. Circles indicate medians and bars indicated interquartile ranges.

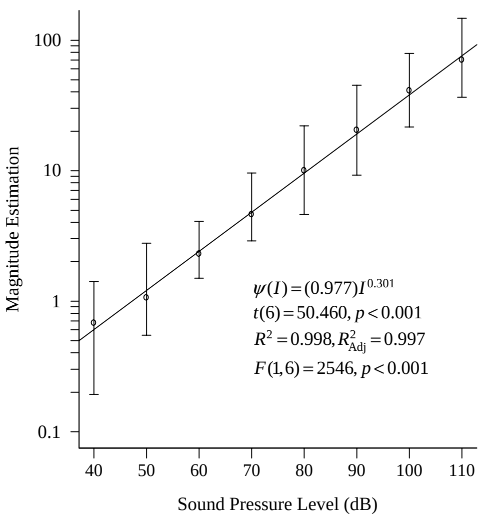

Figure 4. Simulation generated results using an increment of 200. Circles indicate medians and bars indicated interquartile ranges.

variance than the experimental subjects, as indicated by the larger interquartile ranges.

It was observed that while the model exponents produced between simulations (with identical parameters) were always close to each other, there was some variance. However, by increasing the number of simulated participants (using the other simulation parameters employed in the generation of Figure 4), the exponent was observed to converge to approximately 0.28 (Figure 6). With the exception of one point (which was

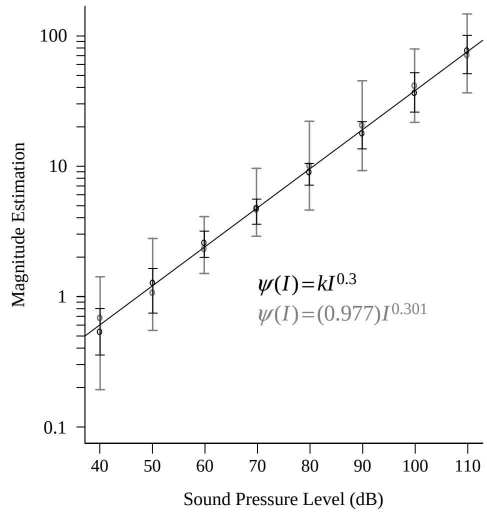

Figure 5. Simulation results from Figure 4 (grey) overlaid on Stevens' [1][3] original results as presented in Figure 2 (black). Circles indicate medians and bars indicated interquartile ranges.

observed during a simulation run with 10 participants) all of the exponents observed rounded to 0.3, the exponent Stevens reported. Since a single decimal point is the standard precision for reporting psychophysical power law exponents [2], this does seem to indicate that the 26 participants simulated in this experiment were sufficient to approximate this particular power law's exponent.

IV. Discussion and Future Work

The simulation model produced a number of interesting results which provide insight into Stevens' power law and open avenues for future work. Through model exploration it was discovered that psychophysical power law data could be generated using a simulation where simulated participants made random ordinal magnitude estimates via a process that mimicked the experimental procedure developed by Stevens. Further, desired exponents could be achieved by changing the constant increment the process used to control the upper bound of ordinal judgments.

Although variation was observed between exponents generated by separate simulations with identical parameters, it was discovered that not only was variation reduced by increasing the number of simulated participants, but that 26 simulated participants (corresponding to Stevens' 26 participants) were capable of producing exponents that were consistent at one decimal place (the precision with which most power law exponents are reported) [2].

While these results do not prove that humans are generating power law data in a manner similar to that used in the simulation, they do show that it is possible to obtain power law data even if participants are making magnitude estimates on an ordinal scale. This supports the assertion that Stevens' power law is an emergent feature of the procedure employed by

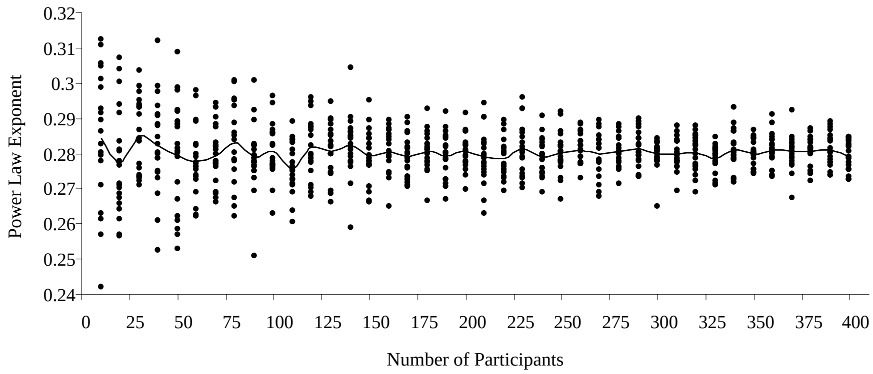

Figure 6. Multiple simulation-generated power law exponents for different numbers of simulated participants.

Stevens rather than an innate aspect of the human sensory system.

Even if the power law is a valid representation of sensation, the results presented here indicate that previous results some using Stevens' procedure could have been derived from ordinal data. If ordinal data are used to compute psychometric power law functions, then those functions will be invalid as the power function will not be modeling perceived magnitude on a ratio scale. Thus, these results suggest that researcher must be extremely careful to collect data only from participants capable of performing magnitude estimation on a ratio scale.

There are a variety of ways the simulation in this experiment could be extended. First, there are a number of other sensory modalities for which power laws have been observed (see [2]). Future work will attempt to replicate these using the simulation procedure described in this manuscript.

Extensions also exist in terms of how the ordinal magnitude estimates are made. The current model arbitrarily assumes human participants would make ordinal magnitude estimates using the method evaluated in the analysis. Thus, a human subject experiment could be conducted in order to determine how humans make ordinal judgments in a magnitude estimation context. Such data could, in turn, be used to develop a more realistic ordinal magnitude estimate generation process. Such a process could also potentially reduce the variance seen in generated data.

Similarly, it has been suggested that, when people make magnitude estimates, they only consider the last stimulus they experienced [4]. As such, future work should incorporate this assumption (and other limits to human memory and cognition) into the simulation in order to see how it effects results.

Work by Teghtsoonian [9] has suggested that when people make magnitude estimates, they do so using a constant judgment range. That is, the difference between a person's maximum magnitude estimate and a person's minimum magnitude estimate is constant irrespective of the sensory modality or range of stimuli used in an experiment. Future work will investigate using this constant range to predict the increment parameter or otherwise influence the way magnitude estimates are generated in the simulation.

Another potential extension of this work can be found in Steven's other psychophysical experimental procedures. Magnitude estimation was only one of three methods Stevens used to obtain power law data. The other two were magnitude production and cross modality matching [2], both variants of the magnitude estimation production. In magnitude production, participants are presented with numbers and are asked to adjust stimulus magnitudes to match the numbers. In cross modality matching, participants were asked to match the intensity of a stimulus in one modality by adjusting the stimulus intensity of another. Future work should construct simulations to replicate these procedures in order to see if ordinal judgments in the context of these procedures could produce power law data.

ACKNOWLEDGEMENT

The author would like to thank Dr. Ellen J. Bass, Dr. Gregory J. Gerling, and Dr. Gerard P. Learmonth Sr. for their assistance.

The project described was supported in part by Grant Number T15LM009462 from the National Library of Medicine and Research Grant Agreement UVA-03-01, sub-award 6073-VA from the National Institute of Aerospace (NIA). The content is solely the responsibility of the authors and does not necessarily represent the official views of the NIA, NASA, the National Library of Medicine, or the National Institutes of Health.

REFERENCES

[1] S. S. Stevens, "The direct estimation of sensory magnitudes-loudness," American Journal of Psychology, 69, 1956, pp. 1-25.

[2] S. S. Stevens, Psychophysics: Method and Theory, John Wiley and Sons, New York, 1976.

[3] S. S. Stevens, "The measurement of loudness," The Journal of the Acoustic Society of America, 27, 1955, pp. 815-829.

[4] D. Laming, "The Measurement of Sensation," New York: Oxford University Press, 1997.

[5] D. M. Green & R. D. Luce, "Variability of magnitude estimates: a timing theory analysis," Perception & Psychophysics, 15, 1974, pp. 291–300.

[6] L. Narens, “A theory of ratio magnitude estimation,” Journal of Mathematical Psychology, 40, 1996, 109–129.

[7] W. Ellermeier & G. Faulhammer, “Empirical evaluation of axioms fundamental to Stevens's ratio-scaling approach: I. Loudness production.,” Perception & Psychophysics, 62, 2000, 1505–1511.

[8] K. Zimmer, “Examining the validity of numerical ratios in loudness fractionation.,” Perception & Psychophysics, 67, 2005, pp. 569–579.

[9] R. Teghtsoonian, “Range effects in psychophysical scaling and a revision of Stevens’s law,” American Journal of Psychology, 86, 1973, pp. 3–27.